# Classifying Stars II: Magnitudes and The Hertzsprung-Russell Diagram

**Apparent magnitude**

You can always trust astronomers to take a simple idea and make it seem complicated. However, once you look at it closely, it may turn out to be the simplest way. The magnitude scale is such a thing.

The magnitude system began harmlessly enough. Scientists like to classify objects, and the Greek astronomer Hipparcos (160-127 B.C.) grouped the visible stars into six classes based on their apparent brightness. The only instrument he had to work with was his eyes, and he figured he could compare two stars and determine whether one was about twice as bright as the other.

The brightest stars he called first-class stars, the next faintest he called second-class stars, and so on (Seeds 1997). The brightness classes are now known as **apparent magnitudes**, and are denoted by a lowercase **m**. The system is counterintuitive in that we naturally associate larger numbers with increasing brightness. The magnitude system uses the reverse philosophy -- a first magnitude star is brighter than a sixth magnitude star. Although the magnitude system can be awkward, it is so deeply ingrained in astronomical literature (and databases) that it would be difficult to abandon the system now. For Hipparcos, magnitude 2 is half as bright as magnitude 1, magnitude 3 is half as bright as magnitude 2 and so ¼ as bright as magnitude 1 and so on. Once Hipparcos got down to magnitude 6, he was looking at the dimmest stars he could make out.

In Atlanta, you are doing good if you can see down to magnitude 3 stars. That’s because of all the lights. Installing lights that point down instead of up would almost eliminate this problem. It would also be cheaper in the long run since none of the light is being wasted by shining upward, all is directed toward the ground where you need it, so you need fewer lights or the same number but less energy consumption for each. This phenomenon is called light pollution and is a serious problem for astronomy. If you’ve never been far from cities, say out in a National Forest, you’ve never seen the sky and you owe it to yourself to make the trip.

As instruments were developed which could measure light levels more accurately than the human eye, and telescopes revealed successively dimmer stars, the magnitude system was refined. A subscript may be added to the apparent magnitude to signify how the magnitude was obtained. The most common magnitude is the **V** magnitude, denoted **mV**, which is obtained instrumentally using an astronomical **V filter**. The V filter allows only light near the wavelength of 550 nanometers (green) to pass through it (approximately 505-595 nm). V magnitudes are very close to those perceived by the human eye since 550 nm is the middle of the visible spectrum. The following chart lists the apparent V magnitudes of a few common celestial objects.

**Table 1: Some common apparent magnitudes (mV)**

|  |  |
| --- | --- |
| **Object** | **mV** |
| Sun | -26.8 |
| Full Moon | -12.5 |
| Venus at brightest | -4.4 |
| Jupiter at brightest | -2.7 |
| Sirius | -1.47 |
| Vega | 0.04 |
| Betelgeuse | 0.41 |
| Polaris | 1.99 |
| Naked eye limit | 6 |
| Pluto | 15.1 |
| Hubble Space Telescope | 31 |

Magnitudes from Seeds (1997), Kaufmann (1996), and Burnham (1978).

While you may perceive one star to be only a few times brighter than another, the intensity of the two stars may differ by orders of magnitude. (Light intensity is defined as the amount of light energy striking each square meter of surface per second.)

A. Take a look at the spreadsheet called Brightest Stars. In the luminosity column, what are the largest and smallest values? Suppose you were to plot these on the axis of a graph – how long would that axis have to be to contain them all? And these are just the brightest stars. There are even more dim stars that have to crowd in between 0 and 1.

The eye is a logarithmic detector. When the eye sees linear steps in brightness, the light intensity is changing by powers. This is fortunate; if the eye responded linearly instead of logarithmically to light intensity, if you could distinguish objects in bright sunlight, you would be nearly blind in the shade! You may not have heard of logarithms in your math classes, but think of them as the exponents. For powers of 10 (so a log base 10), the log of 100 = 102 is 2. The log of 1000 = 103  is 3, and so on.

Given that the eye is a logarithmic detector, and the magnitude system is based on the response of the human eye, it follows that the magnitude system is a logarithmic scale. In the original magnitude system, a difference of 5 magnitudes corresponded to a factor of roughly 100 in light intensity. The magnitude system was later formalized so that a factor of 100 in intensity corresponds exactly to a difference of 5 magnitudes. Since a logarithmic scale is based on multiplicative factors, each magnitude corresponds to a factor of the 5th root of 100, or 2.512, in intensity. The magnitude scale is thus a logarithmic scale in base 1001/5 = 2.512. So instead of powers of 10, the magnitude scale is in powers of 2.512. The following table illustrates the point.

**Table 2: Magnitudes and intensity**

|  |  |
| --- | --- |
| Magnitude difference | Relative intensity |
| 0 | 1 |
| 1 | 2.51 |
| 2 | 6.31 |
| 3 | 15.8 |
| 4 | 39.8 |
| 5 | 100 |
| 10 | 104 |
| 15 | 106 |

To reiterate, each magnitude corresponds to a factor of 2.512 in intensity, and each 5 magnitudes corresponds to a difference in 100 in intensity.

B. Look at the table of brightest stars AND the table of nearest stars. What are the largest and smallest magnitudes in these tables? If you plotted those instead of luminosity, how would that improve your graph?

A few examples will help clarify the point.

C. Star A has an apparent visual magnitude of 7, and its light intensity is 100 times dimmer than that of star B. What is the apparent visual magnitude of star B? Start by finding the magnitude difference.

D. Star A has a magnitude of 2.5, and star B has a magnitude of 4.5. What is the relative intensity IA/IB, where IA is the intensity of star A, and IB is the intensity of star B?

Sometimes the numbers are not this simple, and we need general equations. How can we mathematically describe the relationship between intensity ratios and magnitude differences? While the magnitudes are increasing linearly, the intensity ratios are increasing logarithmically in base 2.512. Denoting magnitudes by **m**, intensities by **I**, and using subscripts **A** and **B** to denote stars A and B, we can express the relationship between intensity and magnitudes as follows:

IA / IB = (2.512)(mB - mA).

Convince yourself that this equation describes the numbers in Table 2. This form of the relationship is best when you know the relative magnitudes, and want to calculate the intensity ratio. We can manipulate the equation to put it in a more convenient form for the opposite case when we know the intensities, but wish to find the relative magnitudes. Taking the log of both sides we get

log10(IA / IB) = log10 (2.512)(mB - mA).

A general rule for logarithms is that log10 MP = p log10 M, so we can rewrite this as

log10(IA / IB) = (mB - mA) log10 2.512

or

log10(IA / IB) = 0.4 (mB - mA).

The result is most commonly expressed in the form

mB - mA = 2.5 log10 (IA / IB).

The red equations are the ones you’ll want in the future. Let's do an example of the magnitude system, this time using the equations.

E. The intensity of star B is a factor of 10 higher than that of star A. Star A has a magnitude of 2.4. What is the magnitude of star A? Let’s do this in steps.

1. First of all, think through the problem intuitively. Which star is brighter? Should B have a higher or a lower magnitude?

2. Use table 2 to estimate the magnitude difference, and then use that to estimate the magnitude of B.

3. Now that you have an idea what is going on, use one of the red equations to calculate the magnitude of B. How does your result compare to your estimate from step 2?

**Absolute magnitude**

Apparent magnitudes describe how bright stars appear to be. However, they tell us nothing about the intrinsic brightness of the stars. Why? The apparent brightness of a star depends on two factors: the intrinsic brightness of the star, and the distance to the star. The Sun is not particularly bright as stars go, but it appears spectacularly bright to us because we live so close to it.

If we want to compare the intrinsic brightness of stars using the magnitude system, we have to level the playing field. We will have to imagine that all the stars are at the same distance, and then measure their apparent brightness. We define the **absolute magnitude** of an object as the apparent magnitude one would measure if the object was viewed from a distance of 10 parsecs (10 pc, where 1 pc = 3.26 light years). We denote absolute magnitude by an upper case **M**. As before, we denote such magnitudes measured through a V filter by the subscript V. The absolute magnitude is thus a measure of the intrinsic brightness of the object. Let's look at the apparent and absolute magnitudes of the stars in Table 2.

**Table 3: Apparent and absolute magnitudes of common stars**

|  |  |  |
| --- | --- | --- |
| **Object** | **mV** | **MV** |
| Sun | -26.8 | 4.83 |
| Sirius | -1.47 | 1.41 |
| Vega | 0.04 | 0.5 |
| Betelgeuse | 0.41 | -5.6 |
| Polaris | 1.99 | -3.2 |

Magnitudes from Seeds (1997) and Burnham (1978).

F. Which of these stars is intrinsically the brightest? Which is intrinsically the dimmest?

How can we calculate the absolute magnitude of a star? One method is to determine the distance to the star, measure the apparent magnitude, and scale the apparent magnitude to a distance of 10 pc. Star distances are a thing we now know how to measure using parallax. The relationship between distance, apparent magnitude and absolute magnitude is the last useful equation we need so lets figure it out.

Check your results with your instructor at this point

**Distance modulus and distance**

How does the apparent magnitude change with distance? We know that a light source becomes fainter as it recedes from us, but can we quanChart, radar chart

Description automatically generatedtify the intensity as a function of distance? Yes!

Imagine a point source that flashes on for one second, then off again. The light pulse spreads out in all directions, traveling at the speed of light. You can think of the light pulse as an expanding shell of light energy. The original light energy in that 1-second pulse is spread over an ever enlarging spherical shell. Intensity is the light energy that strikes 1 square meter in one second. As the shell expands, there is less light energy available for each square cm on the shell as it has to be spread over a larger and larger sphere. At a distance **d** from the original source, the shell has a surface area of (4 x pi x d2). Thus, the intensity **I** decreases as 1/d2. Although this is easiest to picture with a pulse of light, the relationship between distance and intensity is just as true for any source of (incoherent[[1]](#footnote-1)) light. You can think of a continuous source as emitting a series of shells one right after the other. Light intensity decreases as the distance squared. All other things being equal, a light source 3 times more distant is 9 times as faint. For two identical light sources **A** and **B** at distances **dA** and **dB**, the ratio of the intensities are given by

IA / IB = (dB / dA)2,

Now that we know how distance and intensity are related, and how intensity and magnitudes are related, we can determine how distances and magnitudes are related.

In the first section on apparent magnitudes we saw that

mB - mA = 2.5 log10 (IA / IB).

Instead of comparing the intensities and magnitudes of two different stars, we will compare the intensities and magnitudes of the same star at two different distances. Substituting (dB / dA)2 for (IA / IB), we get

mB - mA = 2.5 log10 (dB / dA)2,

which can be rewritten as

mB - mA = 5 log10 (dB / dA).

Now let's look at the special case when dA = 10 pc, so that mA = MA. We can then drop the subscripts A and B, and simply write

m - M = 5 log10 [ d / (10 pc) ].

If we require that **d** be specified in pc, we can rewrite the equation as

m - M = -5 + 5 log10 d.

The quantity **(m - M)** is called the **distance modulus**. Once we know the distance modulus, we can easily calculate the distance to the object. Rewriting the equation as

log10 d = 0.2 (m - M + 5)

and exponentiating both sides, we find that

d = 10 0.2 (m - M + 5),

where **d** is in pc. Assuming no other factors are involved, we now have the explicit relationship between apparent magnitude, absolute magnitude, and distance. Knowing any two allows us to solve for the third. The distance modulus automatically provides two of the unknowns. Table 4 adds distance modulus and approximate distance (in both pc and light years) to the values in Table 3.

**Table 4: mV, MV, distance modulus, and distance for some common stars.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Object** | **mV** | **MV** | **mV-MV** | **d(pc)** | **d(ly)** |
| Sun | -26.8 | 4.83 | -31.6 | 4.72 x 10-6 | 1.53 x 10-5 |
| Sirius | -1.47 | 1.41 | -2.88 | 2.65 | 8.65 |
| Vega | 0.04 | 0.5 | -0.46 | 8.1 | 26 |
| Betelgeuse | 0.41 | -5.6 | 6.0 | 160 | 520 |
| Polaris | 1.99 | -3.2 | 5.2 | 110 | 360 |

Magnitudes from Seeds (1997) and Burnham (1978).

What else might affect the apparent brightness of a star? As we will see in the last section, interstellar dust dims starlight. If a star is far enough away, we must take this dimming into account.

**Interstellar reddening**

Although we think of interstellar space as a vacuum, it is in fact filled with tenuous gas and dust. Like a smoke-filled room, the gas and dust along the line of sight to a star dim the starlight by absorbing and scattering the light. We call this effect **interstellar extinction**. If we do not account for this extinction, we will overestimate the distance to the star.

Extinction is stronger at shorter wavelengths, as shorter wavelengths interact more strongly with dust particles. Red light passes through gas and dust more easily than blue light. The more gas and dust between you and the source, the stronger the reddening. You observe this effect daily! When the sun and moon are near the horizon, you are viewing them through more atmosphere than when they are overhead. That is why the sun and moon look reddish when they rise and set. The reddening of starlight due to the interstellar extinction is known as **interstellar reddening**. Astronomers often used the terms extinction and reddening interchangeably.

The extinction or reddening to an object is usually given in magnitudes, and denoted by an upper case **A**. Since extinction is a function of wavelength, a subscript specifies the wavelength for the stated value. A star whose light is dimmed by 1.2 magnitudes when viewed through a V filter would have an extinction of AV = 1.2.

How do we correct the equation for distance when accounting for extinction? Without extinction, we have

d = 10 0.2 (m - M + 5).

Can you guess how this equation should be altered? We need to brighten the apparent magnitude to correct for the extinction. Lower magnitudes are brighter, so we want to subtract AV from the apparent magnitude. The revised equation is thus

d = 10 0.2 (m - M + 5 - AV),

where d is in pc.

How can you determine AV if you don't know the distance to the star? One trick is to realize that extinction is very low at longer wavelengths. Distances calculated from apparent and absolute magnitudes at long wavelengths will be relatively accurate. The spectral type and luminosity class (something we’ll get to later) of the star determine its absolute magnitude, and also how the stellar emission changes with wavelength. If you can obtain the spectral type and luminosity class of the star, you can then compare the calculated emission at various wavelengths with the corresponding observed emission. The difference between the expected and observed emission at a given wavelength gives you the extinction at that wavelength.

This analysis is a little more complicated than we want to get into, but I thought it was worth mentioning.

## Problems

1. Rigel (Beta Orionis) has an apparent magnitude of +0.14 and Deneb (Alpha Cygni) has an apparent magnitude of+ 1.26. What is the ratio of the brightness of Rigel to Deneb?

2. How many times brighter is the full Moon (-12.5) than Venus at its brightest (-4.6)?

3. How many times dimmer is Venus currently than at its brightest? *Hint* You can use Stellarium to find today’s apparent magnitude for Venus. Just search for it and look at the data table.

4. How many times brighter is Jupiter than Mars, currently?

5. How far away is Canopus (Alpha Carinae), which has an apparent magnitude of -0.72? The spectral and luminosity classes are A9 II.

6. What would be the apparent magnitude of the Sun if it were at the same distance as Sirius (2.7 pc away)? Would you be able to see it from here? The Sun's spectral and luminosity class is G2V.

7. Estimate the distances to each of these stars.

|  |  |  |  |
| --- | --- | --- | --- |
| **Star** | **Spectral Class** | **B – V Color Index** | **Apparent Magnitude** |
| Proxima Centauri | M5 V | 1.82 | 11 |
| Rigel | B8 Ia | -0.03 | 0.13 |
| Betelgeuse | M1 Ia | 1.85 | 0.5 |
| Van Maanan’s Star | DZ8 | 0.546 | 12.374 |
| Arcturus | K1 III | 1.23 | -0.05 |

8. The Hipparcos satellite was able to measure distances using the parallax method out to about 100 pc with 10% accuracy. Except for Rigel and Betelgeuse, these stars are close enough to use that method. For these two stars, you have to use the method here, called spectroscopic parallax, even though it has nothing to do with parallax[[2]](#footnote-2).

1. For the other stars (not Rigel or Betelgeuse), look up the distances determined by parallax. Wikipedia is a good place to go. How would you assess the accuracy of spectroscopic parallax? Given the steps in the method, where do you think most of the uncertainty comes from?
2. Many stars come in clusters, groups of stars that are all at about the same distance. For example, the Pleiades open cluster contains about 1000 stars. Globular clusters (groups of stars outside the galaxy but associated with it) may contain tens to hundreds of thousands of stars. Why would spectroscopic parallax be a much better method of estimating the distance to clusters than to individual stars?

1. So excluding things like lasers. Incoherent means several things but part of it is “emitted equally in all directions.” [↑](#footnote-ref-1)
2. There are certain types of variable stars for which a much better method can be used as there is a relationship between the time for brightness variation and the luminosity. However, these are relatively rare as they are a transient phase in the lifetime of stars. [↑](#footnote-ref-2)